Swaps
A swap is an OTC agreement between two companies to exchange cash flows in the future.

The agreement defines the dates when the cash flows are to be paid and the way in which they are to be calculated.

Usually the calculation of the cash flows involves the future value of an interest rate, or exchange rate, or some other market variable.

Bonds, forward rate agreements, and interest rate futures are key examples of the instruments that can be used for hedging in swap markets.
The most common type of swap is the “plain vanilla” interest rate swap. In this swap, a company agrees to pay cash flows equal to interest at a predetermined fixed rate on a notional principal for a predetermined number of years. In return, the company receives interest at a floating rate on the same notional principal for the same period of time.
Consider a 3-year swap initiated on March 5, 2012, between Microsoft and Intel.

Microsoft agrees to pay Intel an interest rate of 5% per annum on a notional principal of $100 million, and in return Intel agrees to pay Microsoft the 6-month LIBOR rate on the same principal.

Microsoft is the fixed-rate payer; Intel is the floating-rate payer.

The agreement specifies that payments are to be exchanged every 6 months and that the 5% interest rate is quoted with semiannual compounding.
The first exchange of payments will take place on September 5, 2012, 6 months after the initiation of the agreement.

Microsoft will pay Intel $2.5 million. This is the interest on the $100 million principal for 6 months at 5% annual rate.

Intel will pay Microsoft interest on the $100 million principal at the 6-month LIBOR rate prevailing 6 months prior to September 5, 2012—that is, on March 5, 2012.
Suppose that the 6-month LIBOR rate on March 5, 2012, is 4.2%.

Intel pays Microsoft $2.1 million.

Note that we ignore day count conventions for simplicity.

There is no uncertainty about this first exchange of payments because it is determined by the LIBOR rate at the time the contract is entered into.
The second exchange of payments would take place on March 5, 2013, a year after the initiation of the agreement.

Microsoft would pay $2.5 million to Intel. Intel would pay interest on the $100 million principal to Microsoft at the 6-month LIBOR rate prevailing 6 months prior to March 5, 2013—that is, on September 5, 2012.

Suppose that the 6-month LIBOR rate on September 5, 2012, is 4.8%.

Intel pays: \(0.5 \times 0.048 \times $100 = $2.4\) million to Microsoft.
In total, there are six exchanges of payment on the swap.

The fixed payments are always $2.5 million.

The floating-rate payments on a payment date are calculated using the 6-month LIBOR rate prevailing 6 months before the payment date.

An interest rate swap is structured so that one side remits the difference between the two payments to the other side.

Microsoft will pay Intel ($2.5 million - $2.1 million) on September 5, 2012, and ($2.5 million - $2.4 million) on March 5, 2013 and so on.
"Plain Vanilla" Interest Rate Swaps

Cash flows ($ millions) to Microsoft in a $100 million 3-year interest rate swap when a fixed rate of 5% is paid and LIBOR is received.

<table>
<thead>
<tr>
<th>Date</th>
<th>Six-month LIBOR rate (%)</th>
<th>Floating cash flow received</th>
<th>Fixed cash flow paid</th>
<th>Net cash flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mar. 5, 2012</td>
<td>4.20</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sept. 5, 2012</td>
<td>4.80</td>
<td>+2.10</td>
<td>−2.50</td>
<td>−0.40</td>
</tr>
<tr>
<td>Mar. 5, 2013</td>
<td>5.30</td>
<td>+2.40</td>
<td>−2.50</td>
<td>−0.10</td>
</tr>
<tr>
<td>Sept. 5, 2013</td>
<td>5.50</td>
<td>+2.65</td>
<td>−2.50</td>
<td>+0.15</td>
</tr>
<tr>
<td>Mar. 5, 2014</td>
<td>5.60</td>
<td>+2.75</td>
<td>−2.50</td>
<td>+0.25</td>
</tr>
<tr>
<td>Sept. 5, 2014</td>
<td>5.90</td>
<td>+2.80</td>
<td>−2.50</td>
<td>+0.30</td>
</tr>
<tr>
<td>Mar. 5, 2015</td>
<td>5.90</td>
<td>+2.95</td>
<td>−2.50</td>
<td>+0.45</td>
</tr>
</tbody>
</table>
Note that the $100 million principal is basically used for the calculation of interest payments.

Normally, the principal itself is not exchanged.

If the principal were exchanged at the end of the life of the swap, the nature of the deal would not be changed in any way.
“Plain Vanilla” Interest Rate Swaps

Cash flows ($ millions) to Microsoft in a $100 million 3-year interest rate swap when a fixed rate of 5% is paid and LIBOR is received and when there is a final exchange of principal.

<table>
<thead>
<tr>
<th>Date</th>
<th>Six-month LIBOR rate (%)</th>
<th>Floating cash flow received</th>
<th>Fixed cash flow paid</th>
<th>Net cash flow</th>
</tr>
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<tr>
<td>Mar. 5, 2012</td>
<td>4.20</td>
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<td>5.90</td>
<td>+2.80</td>
<td>−2.50</td>
<td>+0.30</td>
</tr>
<tr>
<td>Mar. 5, 2015</td>
<td></td>
<td>+102.95</td>
<td>−102.50</td>
<td>+0.45</td>
</tr>
</tbody>
</table>
The “plain vanilla” interest rate swap can be regarded as the exchange of a fixed-rate bond for a floating-rate bond.

The cash flows in the third column of the tables above are the cash flows from a long position in a floating-rate bond.

The cash flows in the fourth column of the tables are the cash flows from a short position in a fixed-rate bond.

Microsoft is long a floating-rate bond and short a fixed-rate bond.

Intel is long a fixed-rate bond and short a floating-rate bond.
The Role of Financial Intermediaries

- Usually two nonfinancial companies do not get in touch directly to arrange a swap in the way described before.
- They each deal with a financial intermediary such as a commercial bank.
- “Plain vanilla” fixed-for-floating swaps on interest rates are usually structured so that the financial intermediary is expected to earn about 0.03% or 0.04% on a pair of offsetting swap transactions.
- For 0.03%, the intermediary makes a profit/year of:
  \[(0.03\% \times L)\]
  where \(L\) is the notional principal.
- In our example, this amounts to $30,000 per year for the 3-year period.
The financial intermediary has two separate contracts: one with Intel and one with Microsoft. The one firm does not necessarily know that the intermediary has entered into an offsetting swap with some other firm. If one of the companies defaults, the financial intermediary has to honour its agreement with the other company. The spread of 0.03% earned by the financial intermediary is partly to compensate it for the risk of default on the swap payments in any of the two parties. The cost of $30,000 per year will equally burden both companies.
A swap can be characterised as the difference between a fixed-rate bond and a floating-rate bond, which provide us with the bid-offer spread:

\[ B_{\text{fix}} - B_{\text{fl}} \]  

(1)

where:

- \( B_{\text{fix}} \) is the value of the fixed-rate bond underlying the swap under examination
- \( B_{\text{fl}} \) is the value of floating-rate bond underlying the swap under examination

The average of the bid and offer fixed rates is known as the swap rate.
Swap rates define a set of par yield bonds.

The par yield for a certain bond maturity is the coupon rate that causes the bond price to equal its par value.

The par value is the same as the principal value.
Derivatives traders traditionally use LIBOR rates as proxies for risk-free rates when valuing derivatives.

One problem with LIBOR rates is that direct observations are possible only for maturities out to 12 months.

Traders use swap rates to extend the LIBOR zero curve further.

The resulting zero curve is sometimes referred to as the LIBOR/swap zero curve.

We will now describe how swap rates are used in the determination of the LIBOR/swap zero curve.
The first point to note is that the value of a newly issued floating-rate bond that pays 1-month, or 3-month, or 6-month, or 12-month LIBOR is always equal to its principal value (or par value) when the LIBOR/swap zero curve is used for discounting.

The reason is that the bond provides a rate of interest of LIBOR, and LIBOR is also the discount rate.

Therefore, the interest on the bond exactly matches the discount rate when the bond is issued, and as a result the bond is fairly priced at par.

Since the swap is worth zero, it follows that:

\[ B_{fix} - B_{fl} = 0 \]
Determination of LIBOR/Swap zero rates

- $B_{fix} - B_{fl} = 0$ implies that $B_{fix}$ and $B_{fl}$ are equal to each other and they are both equal to the notional principal.
- Hence, as previously mentioned, swap rates define a set of par yield bonds.
- For example, following the above table, we can deduce that the:
  - 2-year LIBOR/swap par yield is 6.045%
  - 3-year LIBOR/swap par yield is 6.225%
  - and so on…
Suppose that the 6-month, 12-month, and 18-month LIBOR/swap zero rates have been determined as 4%, 4.5%, and 4.8% with continuous compounding.

The 2-year swap rate (for a swap where payments are made semiannually) is 5%.

This 5% swap rate means that a bond with a principal of $100 and a semiannual coupon of 5% per annum sells for par.

We assume a bond with a principal of $100 and we wish to calculate $R$, where $R$ is the 2-year LIBOR/swap zero rate.

Following the typical bond pricing method, we calculate:

$$2.5e^{-0.04 \times 0.5} + 2.5e^{-0.045 \times 1.0} + 2.5e^{-0.048 \times 1.5} + 102.5e^{-2R} = 100$$

We obtain $R = 4.953\%$. Note that this calculation does not take the swap’s day count conventions into account.
The day count conventions affect payments on a swap.
Consider, for example, the 6-month LIBOR payments in the Tables in the example with Microsoft and Intel.
Because both are US companies, the 6-month LIBOR in the US money markets is quoted on an actual/360 basis.
The first floating payment based on the LIBOR rate of 4.2%, is shown as $2.10 million.
Because there are 184 days between March 5, 2012, and September 5, 2012, it should be

\[
100 \times 0.042 \times \frac{184}{360} = \$2.1467 \text{ million}
\]
In general, a LIBOR-based floating-rate cash flow on a swap payment date is calculated as:

\[ L \times R \times \frac{\text{actual days}}{360} \]

where \( L \) is the principal, and \( R \) is the relevant zero rate.
Valuation of Interest Rate Swaps

- An interest rate swap is equal to zero when it is first initiated (in a new swap, the fixed rate equals the current swap rate).
- After it has been in existence for some time, its value may be positive or negative.
- There are two valuation approaches.
  - The first regards the swap as the difference between two bonds.
  - The second regards it as a portfolio of Forward Rate Agreements (FRAs).
From the point of view of the floating-rate payer (Intel), a swap can be regarded as a long position in a fixed rate bond and a short position in a floating-rate bond, so that:

\[ V_{swap} = B_{fix} - B_{fl} \]

Similarly, from the point of view of the fixed-rate payer (Microsoft), a swap is a long position in a floating-rate bond and a short position in a fixed-rate bond, so that the value of the swap is:

\[ V_{swap} = B_{fl} - B_{fix} \]
Valuation in Terms of Bond Prices

- The value of $B_{fix}$ is determined using the standard bond pricing method.
- To value $B_{fl}$, we argue that the bond is equal to $L$ immediately after an interest payment: $B_{fl} = L$
- This is because at this time the bond is a “fair deal” where the borrower pays LIBOR for each subsequent accrual period.
- Suppose that the next exchange of payments is at time $t^*$, and the floating payment that will be made at time $t^*$ is $k^*$.
Immediately before the payment $B_{fl} = L + k^*$

The floating-rate bond can therefore be regarded as an instrument providing a single cash flow of $(L + k^*)$ at time $t^*$.

Discounting this, the value of the floating-rate bond today is $(L + k)e^{-r^*t^*}$, where $r^*$ is the LIBOR/swap zero rate for a maturity of $t^*$. 

Valuation in Terms of Bond Prices
Suppose that a financial institution has agreed to pay 6-month LIBOR and receive 8% per annum (with semiannual compounding) on a notional principal of $100 million.

The swap has a remaining life of 1.25 years.

The LIBOR rates with continuous compounding for 3-month, 9-month, and 15-month maturities are 10%, 10.5%, and 11%, respectively.

The 6-month LIBOR rate at the last payment date was 10.2% (with semiannual compounding).
Valuation in Terms of Bond Prices: Example

If the financial institution had been in the opposite position of paying fixed and receiving floating, the value of the swap would be +$4.2671 million.

Note that these calculations do not take account of day count conventions and holiday calendars.
Valuation in Terms of FRAs

- Consider the swap between Microsoft and Intel.
- The swap is a 3-year deal entered into on March 5, 2012, with semiannual payments.
- The first exchange of payments is known at the time the swap is negotiated.
- The other five exchanges can be regarded as FRAs.
- The exchange on March 5, 2013, is a FRA where interest at 5% is exchanged for interest at the 6-month rate observed in the market on September 5, 2012.
- The exchange on September 5, 2013, is a FRA where interest at 5% is exchanged for interest at the 6-month rate observed in the market on March 5, 2013.
A FRA can be valued by assuming that forward interest rates are realised.

Forward interest rates are the rates of interest implied by current zero rates for periods of time in the future.

The rates are assumed to be continuously compounded.

The 3% per annum rate for 1 year means that, in return for an investment of $100 today, an amount $100e^{0.03 \times 1} = $103.05 is received in 1 year.

The 4% per annum rate for 2 years means that, in return for an investment of $100 today, an amount $100e^{0.04 \times 2} = $108.33 is received in 2 years.

And so on...
Because it is nothing more than a portfolio of forward rate agreements, a plain vanilla interest rate swap can also be valued by making the assumption that forward interest rates are realised.

The procedure is as follows:

a) Use the LIBOR/swap zero rates to calculate forward rates for each of LIBOR rates that will determine swap cash flows.

b) Calculate swap cash flows on the assumption that the LIBOR rates will equal the forward rates.

c) Discount these swap cash flows (using the LIBOR/swap zero curve) to obtain the swap value.
We resort to the data used in the valuation in terms of bond prices.

The first row shows the cash flows that will be exchanged in 3 months. These have already been determined.

The fixed rate of 8% leads to a cash inflow of $4 million.

The floating rate of 10.2% (which was set 3 months ago) will lead to a cash outflow of $5.1 million.

### Valuation in Terms of FRAs: Example

<table>
<thead>
<tr>
<th>Time</th>
<th>Fixed cash flow</th>
<th>Floating cash flow</th>
<th>Net cash flow</th>
<th>Discount factor</th>
<th>Present value of net cash flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>4.0</td>
<td>-5.100</td>
<td>-1.100</td>
<td>0.9753</td>
<td>-1.073</td>
</tr>
<tr>
<td>0.75</td>
<td>4.0</td>
<td>-5.522</td>
<td>-1.522</td>
<td>0.9243</td>
<td>-1.407</td>
</tr>
<tr>
<td>1.25</td>
<td>4.0</td>
<td>-6.051</td>
<td>-2.051</td>
<td>0.8715</td>
<td>-1.787</td>
</tr>
<tr>
<td>Total:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-4.267</td>
</tr>
</tbody>
</table>
The second row shows the cash flows that will be exchanged in 9 months assuming that forward rates are realised.

The cash inflow is $4.0 million as before.

To calculate the cash outflow, we must first calculate the forward rate corresponding to the period between 3 and 9 months.

If $R_1$ and $R_2$ are the zero rates for maturities $T_1$ and $T_2$ respectively and $R_c$ is the forward interest rate for the period of time between $T_1$ and $T_2$, then:

$$R_c = \frac{R_2 T_2 - R_1 T_1}{T_2 - T_1} \quad (2)$$
From the equation (2) and by assuming continuous compounding, we obtain: 
$$ R_c = 0.1075 \text{ or } 10.75\% $$

If $R_c$ is the rate of interest with continuous compounding then $R_m$ is the equivalent rate with compounding $m$ times per annum:

$$ R_m = m(e^{R_c/m} - 1) \quad (3) $$

For $m = 2$ (semiannual compounding), we get that:

$$ R_m = 0.11044 \text{ or } 11.044\% $$

The cash outflow is therefore $0.5 \times 0.11044 \times 100 = 5.522\text{million}$. 

The third row similarly shows the cash flows that will be exchanged in 15 months assuming that forward rates are realised.
The discount factors remain the same as in the case of bond pricing.

The present values of the FRAs corresponding to the exchanges in 3, 9, and 15 months are the same with the present values of net cash flows for the respective maturities.

The total value of the swap is the same as in the case of bond pricing: -$4.267 million
A currency swap involves exchanging principal and interest payments in one currency for principal and interest payments in another.

A currency swap agreement requires the principal to be specified in each of the two currencies.

The principal amounts are usually exchanged at the beginning and at the end of the life of the swap.

Usually the principal amounts are chosen to be approximately equivalent using the exchange rate at the swap’s initiation.

When they are exchanged at the end of the life of the swap, their values may be quite different.
Consider a hypothetical 5-year currency swap agreement between IBM and British Petroleum entered into on February 1, 2011.

We suppose that IBM pays a fixed rate of interest of 5% in sterling and receives a fixed rate of interest of 6% in dollars from British Petroleum.

Interest rate payments are made once a year and the principal amounts are $18 million and £10 million.

This is termed a fixed-for-fixed currency swap because the interest rate in each currency is at a fixed rate.
Currency Swaps: Illustration

- At the outset of the swap, IBM pays $18 million and receives £10 million.
- Each year during the life of the swap contract, IBM receives $1.08 million (6% of $18 million) and pays £0.50 million (5% of £10 million).
- At the end of the life of the swap, it pays a principal of £10 million and receives a principal of $18 million.

<table>
<thead>
<tr>
<th>Date</th>
<th>Dollar cash flow (millions)</th>
<th>Sterling cash flow (millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>February 1, 2011</td>
<td>-18.00</td>
<td>+10.00</td>
</tr>
<tr>
<td>February 1, 2012</td>
<td>+1.08</td>
<td>-0.50</td>
</tr>
<tr>
<td>February 1, 2013</td>
<td>+1.08</td>
<td>-0.50</td>
</tr>
<tr>
<td>February 1, 2014</td>
<td>+1.08</td>
<td>-0.50</td>
</tr>
<tr>
<td>February 1, 2015</td>
<td>+1.08</td>
<td>-0.50</td>
</tr>
<tr>
<td>February 1, 2016</td>
<td>+19.08</td>
<td>-10.50</td>
</tr>
</tbody>
</table>
If we define $V_{swap}$ as the value in US dollars of an outstanding swap where dollars are received and a foreign currency is paid, then:

$$V_{swap} = B_D - S_0 B_F$$

where:

$B_D$ is the value of the bond defined by the domestic cash flows on the swap

$B_F$ is the value, measured in the foreign currency, of the bond defined by the foreign cash flows on the swap

$S_0$ is the spot exchange rate (expressed as number of dollars per unit of foreign currency)
Similarly, the value of a swap where the foreign currency is received and dollars are paid is:

$$V_{\text{swap}} = S_0 B_F - B_D$$

- The value of a swap can be determined from LIBOR rates in the two currencies, the term structure of interest rates in the domestic currency, and the spot exchange rate.
- The term structure of interest rates can be flat, upward-sloping, or downward-sloping.
It is natural to ask what determines the shape of the zero curve.

Why is it sometimes downward sloping, and sometimes upward sloping?

Expectations theory conjectures that long-term interest rates should reflect expected future short-term interest rates.

More precisely, it argues that a forward interest rate corresponding to a certain future period is equal to the expected future zero interest rate for that period.

Suppose that the term structure of interest rates is upward-sloping at the time the swap is negotiated.

This means that the forward interest rates increase as the maturity of swap increases.