Hedging Strategies using Futures
Cross Hedging

- The asset that gives rise to the hedger’s exposure is sometimes different from the asset underlying the futures contract that is used for hedging.
- This is known as *cross hedging*.
- Cross hedging usually leads to an increase in basis risk.
- E.g., an airline that is concerned about the future price of jet fuel.
- Because jet fuel futures are not actively traded, the airline might choose to use heating oil futures contracts to hedge its exposure.
Cross Hedging – Hedge Ratio

- The *hedge ratio* is the ratio of the size of the position taken in futures contracts to the size of the exposure.
- When the asset underlying the futures contract is the same as the asset being hedged, a hedge ratio of 1.0 is commonly used. (This is the hedge ratio we have implicitly used in the examples considered so far.)
- When cross hedging is used, hedge ratio is normally different than 1.0
- The hedger chooses a value for the hedge ratio that minimises the variance of the value of the hedged position.
The Minimum Variance Hedge Ratio

- The *variance hedge ratio* depends on the relationship between:
  a) changes in the spot price and
  b) changes in the futures price.
- The *optimal (minimum) variance hedge ratio* is the product of the coefficient of correlation between $\Delta S$ on $\Delta F$ and the ratio of the standard deviation of $\Delta S$ on $\Delta F$.

$$h^* = \rho \frac{\sigma_S}{\sigma_F}$$
The Minimum Variance Hedge Ratio

The figure shows that the variance of the value of the hedger’s position depends on the hedge ratio chosen.
The parameters $\rho$, $\sigma_F$, and $\sigma_S$, in equation are usually estimated from historical data on $\Delta S$ and $\Delta F$.

The assumption is that the future will in some sense be like the past.

A number of equal non-overlapping time intervals are chosen, and the values of $\Delta S$ and $\Delta F$ for each of the intervals are observed.

Ideally, the length of each time interval is the same as the length of the time interval for which the hedge is in effect.

In practice, this limits the number of observations that are available, and a shorter time interval is used.
It can be shown that $h^*$ is the slope of the best-fit line from a linear regression of $\Delta S$ on $\Delta F$. 
Hedge Effectiveness

- The *hedge effectiveness* can be defined as the proportion of the variance that is eliminated by hedging.

- Hedge effectiveness is the coefficient of determination of the regression of $\Delta S$ against $\Delta F$.

- The coefficient of determination shows how well data points fit a statistical model.
The optimal number of contracts reflects the number of futures contracts that should be chosen and used in hedging.

The formula is as follows:

\[ N^* = \frac{h^* Q_A}{Q_F} \]
Optimal Number of Contracts: Example

- An airline expects to purchase 2m gallons of jet fuel in 1 month and decides to use heating oil futures for hedging.
- Each heating oil contract is on 42,000 gallons of heating oil.
- The following table gives, for 15 successive months, data on:
  - $\Delta S$ (changes in the jet fuel price per gallon), and
  - $\Delta F$ (changes in the futures price for the contract on heating oil that would be used for hedging price changes during the month).
Optimal number of contracts: Example

<table>
<thead>
<tr>
<th>$i$</th>
<th>$\Delta F$</th>
<th>$\Delta S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.021</td>
<td>0.029</td>
</tr>
<tr>
<td>2</td>
<td>0.035</td>
<td>0.020</td>
</tr>
<tr>
<td>3</td>
<td>-0.046</td>
<td>-0.044</td>
</tr>
<tr>
<td>4</td>
<td>0.001</td>
<td>0.008</td>
</tr>
<tr>
<td>5</td>
<td>0.044</td>
<td>0.026</td>
</tr>
<tr>
<td>6</td>
<td>-0.029</td>
<td>-0.019</td>
</tr>
<tr>
<td>7</td>
<td>-0.026</td>
<td>-0.010</td>
</tr>
<tr>
<td>8</td>
<td>-0.029</td>
<td>-0.007</td>
</tr>
<tr>
<td>9</td>
<td>0.048</td>
<td>0.043</td>
</tr>
<tr>
<td>10</td>
<td>-0.006</td>
<td>0.011</td>
</tr>
<tr>
<td>11</td>
<td>-0.036</td>
<td>-0.036</td>
</tr>
<tr>
<td>12</td>
<td>-0.011</td>
<td>-0.018</td>
</tr>
<tr>
<td>13</td>
<td>0.019</td>
<td>0.009</td>
</tr>
<tr>
<td>14</td>
<td>-0.027</td>
<td>-0.032</td>
</tr>
<tr>
<td>15</td>
<td>0.029</td>
<td>0.023</td>
</tr>
</tbody>
</table>
Tailing the hedge

- When futures are used for hedging, a small adjustment, known as *tailing the hedge*, can be made to allow for the impact of daily settlement.

- The effect of tailing the hedge is to multiply the hedge ratio by the ratio of the spot price to the futures price.

- Theoretically, the futures position used for hedging should be adjusted as the spot price and futures price change, but in practice this usually makes little difference.

- If forward contracts rather than futures contracts are used, there is no daily settlement.
A *stock index* tracks changes in the value of a hypothetical portfolio of stocks. The weight of a stock in the portfolio at a particular time equals the proportion of the hypothetical portfolio invested in the stock at that time. The % increase in the stock index over a small interval of time is set equal to the % increase in the value of the hypothetical portfolio. Dividends are usually not included in the calculation so that the index tracks the capital gain/loss from investing in the portfolio.
Stock Index Futures

- If the hypothetical portfolio of stocks remains fixed, the weights assigned to individual stocks in the portfolio do not remain fixed.
- When the price of one particular stock in the portfolio rises more sharply than others, more weight is automatically given to that stock.
- Sometimes indices are constructed from a hypothetical portfolio consisting of one of each of a number of stocks.
- The weights assigned to the stocks are then proportional to their market prices, with adjustments being made when there are stock splits.
Other indices are constructed so that weights are proportional to market capitalization (stock price number of shares outstanding).

The underlying portfolio is then automatically adjusted to reflect stock splits, stock dividends, and new equity issues.
Hedging an Equity Portfolio

- Stock index futures can be used to hedge a well-diversified equity portfolio.
- If the portfolio mirrors the index, the optimal hedge ratio, $h^*$, equals 1.0 and the number of futures contracts that should be shorted is given by:

$$N^* = \frac{V_A}{V_F}$$
If the portfolio does not mirror the index, we can use the CAPM.

\[ N^* = \beta \frac{V_A}{V_F} \]

We notice that they \( h^* = \beta \) if tailing-the-hedge formula is considered.

This happens because \( h \) is the slope of the best-fit line when changes in the portfolio are regressed against changes in the futures price of the index.

Beta (\( \beta \)) is the slope of the best-fit line when the return from the portfolio is regressed against the return for the index.
CAPM can be used to calculate the expected return from an asset during a period in terms of the risk of the return.

The risk in the return from an asset is divided into two parts:

a) Systematic risk: related to the return from the market as a whole and cannot be diversified away.

b) Nonsystematic (idiosyncratic) risk is unique to the asset and can be diversified away by choosing a large portfolio of different assets.
The return from the portfolio of all available investments, $R_M$, is referred to as the return on the market and is usually approximated as the return on a well-diversified stock index such as the S&P 500.

The beta ($\beta$) of an asset is a measure of the sensitivity of its returns to returns from the market.

$\beta$ can be estimated from historical data as the slope obtained when the excess return on the asset over the risk-free rate is regressed against the excess return on the market over the risk-free rate.
Hedging an Equity Portfolio & CAPM

- When $\beta=1$, the expected return on the asset equals to the return on the market.
- A portfolio with a $\beta$ of 2.0 is twice as sensitive to movements in the index as a portfolio with a $\beta$ of 1.0.
- It is therefore necessary to use twice as many contracts to hedge the portfolio.
- Similarly, a portfolio with a $\beta$ of 0.5 is half as sensitive to market movements as a portfolio with a $\beta$ of 1.0 and we should use half as many contracts to hedge it.
The derivation of CAPM requires a number of assumptions:

1. Investors care only about the expected return and standard deviation of the return from an asset.
2. The returns from two assets are correlated with each other only because of their correlation with the return from the market.
3. Investors focus on returns over a single period and that period is the same for all investors.
4. Investors can borrow and lend at the same risk-free rate.
5. Tax does not influence investment decisions.
6. All investors make the same estimates of expected returns, standard deviations of returns, and correlations between returns.
Hedging an Equity Portfolio & CAPM

- These assumptions are at best only approximately true.
- Nevertheless CAPM has proved to be a useful tool for portfolio managers and is often used as a benchmark for assessing their performance.
- When the asset is an individual stock, the expected return is not a particularly good predictor of the actual return.
- But, when the asset is a well diversified portfolio of stocks, it is a much better predictor.
- $\beta$ is then calculated as the weighted average of the betas of the stocks in the portfolio.
Reasons for Hedging an Equity Portfolio

- The hedging procedure results in a value for the hedger’s position at the end of the 3-month period being about 1% higher than at the beginning of the 3-month period.
- The hedge results in the investor’s position growing at the risk-free rate.
- Why the hedger should go to the trouble of using futures contracts?
Hedging can be justified if the hedger feels that the stocks in the portfolio have been chosen well.

The hedger might be uncertain about the performance of the market as a whole, but confident that the stocks in the portfolio will outperform the market.

A hedge using index futures removes the risk arising from market moves and leaves the hedger exposed only to the performance of the portfolio relative to the market.

In our example, the S&P 500 falls by 10% whereas the total value of the hedger’s position in 3 months increases.
The expiration date of the hedge might be later than the delivery dates of all the futures contracts that can be used.

The hedger must then roll the hedge forward by closing out one futures contract and taking the same position in a futures contract with a later delivery date.

Hedges can be rolled forward many times.

The procedure is known as *stack and roll*. 
Consider a company that wishes to use a short hedge to reduce the risk associated with the price to be received for an asset at time $T$.

If there are futures contracts 1, 2, 3, . . . , $n$ with progressively later delivery dates, the company can use the following strategy:
Stack and Roll: Example

- Time $t_1$: Short futures contract 1
- Time $t_2$: Close out futures contract 1
  Short futures contract 2
- Time $t_3$: Close out futures contract 2
  Short futures contract 3
- ..............................................................
- Time $t_n$: Close out futures contract $n-1$
  Short futures contract $n$
- Time $T$: Close out futures contract $n$
Table 3.4  Performance of stock index hedge.

<table>
<thead>
<tr>
<th></th>
<th>900</th>
<th>950</th>
<th>1,000</th>
<th>1,050</th>
<th>1,100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of index in three months:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Futures price of index today:</td>
<td>1,010</td>
<td>1,010</td>
<td>1,010</td>
<td>1,010</td>
<td>1,010</td>
</tr>
<tr>
<td>Futures price of index</td>
<td>902</td>
<td>952</td>
<td>1,003</td>
<td>1,053</td>
<td>1,103</td>
</tr>
<tr>
<td>in three months:</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Gain on futures position ($)</td>
<td>810,000</td>
<td>435,000</td>
<td>52,500</td>
<td>−322,500</td>
<td>−697,500</td>
</tr>
<tr>
<td>Return on market:</td>
<td>−9.750%</td>
<td>−4.750%</td>
<td>0.250%</td>
<td>5.250%</td>
<td>10.250%</td>
</tr>
<tr>
<td>Expected return on portfolio:</td>
<td>−15.125%</td>
<td>−7.625%</td>
<td>−0.125%</td>
<td>7.375%</td>
<td>14.875%</td>
</tr>
<tr>
<td>Expected portfolio value in three</td>
<td>4,286,187</td>
<td>4,664,937</td>
<td>5,043,687</td>
<td>5,422,437</td>
<td>5,801,187</td>
</tr>
<tr>
<td>months including dividends ($)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total value of position</td>
<td>5,096,187</td>
<td>5,099,937</td>
<td>5,096,187</td>
<td>5,099,937</td>
<td>5,103,687</td>
</tr>
<tr>
<td>in three months ($)</td>
<td></td>
<td></td>
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</table>